San Diego County Office of Education
College and Career Readiness

Mathematics

Tasks and Talk - Keys to Success in Common Core Mathematics K - 5

San Diego County Office of Education

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Preparing to Learn Together

Welcome!

Outcomes:

- Empowering teachers to make decisions regarding math task selection
- Understand qualities and rationale of using high-level, rich tasks
- Modify tasks in the curriculum
- Create high-level, rich tasks

Smarter Balanced Practice Test

3rd grade Performance Task: Lemonade Stand

Work Space:

Qualities that differentiate Task #4 from Task # 1-3:

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Cognitive Demands of Mathematical Instructional Tasks

Lower-Level Demand

<u>"Memorization" Tasks</u>

- Involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous - such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules formulae, or definitions being learned or reproduced.

Example: What are the decimal and percent equivalents for the fractions $\frac{1}{2}$ and $\frac{1}{4}$?

Expected Student Response:

$$\frac{1}{2} = .5 = 50\%$$
$$\frac{1}{4} = .25 = 25\%$$

<u>"Procedures Without Connections" Tasks</u>

- Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers rather than developing mathematical understanding.
- Require no explanations, or explanations that focus solely on describing the procedure that was used.

Higher-Level Demand

<u>"Procedures With Connections" Tasks</u>

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.

<u>"Doing Mathematics" Tasks</u>

- Require complex and nonalgorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).
- Require students to explore and understand the nature of mathematical concepts, processes, or relationship[s.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through thee task.
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Stein, M. & Smith, M. (1998) Mathematics Teaching in the Middle School. National Council of Teachers of Mathematics

Cognitive Demands of Mathematical Instructional Tasks

Lower-Level Demand

“Memorization” Tasks

What are the decimal and percent equivalents for the fractions $\frac{1}{2}$ and $\frac{1}{4}$?

Expected Student Response:

$\frac{1}{2} = .5 = 50\%$

$\frac{1}{4} = .25 = 25\%$

“Procedures Without Connections” Tasks

Convert the fraction $\frac{3}{8}$ to a decimal and a percent.

Expected Student Response:

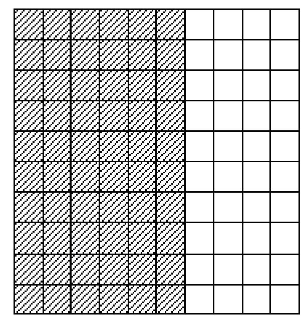
Fraction	*Decimal*	*Percent*
$\frac{3}{8}$	.375 8)3.000 24 60 56 40 40 0	.375 = 37.5%

Higher-Level Demand

“Procedures With Connections” Tasks

Using a 10 x 10 grid, identify the decimal and percent equivalents of $\frac{3}{5}$.

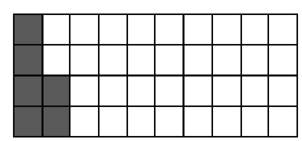
Expected Student Response:

Pictorial	Fraction	Decimal	Percent
	$\frac{60}{100} = \frac{3}{5}$	$\frac{60}{100} = .60$	.60 = 60%

“Doing Mathematics” Tasks

Shade 6 small squares in a 4 x 10 rectangle. Using the rectangle, explain how to determine each of the following: a) the percent of area that is shaded, b) the decimal part of the area that is shaded, and c) the fractional part of area that is shaded.

One Possible Student Response:



a) On column will be 10% since there are 10 columns. So four squares is 10%. Then 2 squares is half a column and half of 10% which is 5%. So the 6 shaded blocks equal 10% plus 5% or 15%.

b) One column will be .10 since there are 10 columns. The second column has only 2 squares shaded so that would be one half of .10 which is .05. so the 6 shaded blocks equal .1 plus .05 which equals .15.

c) Six shaded squares out of 40 squares is $\frac{6}{40}$ which is equivalent to $\frac{3}{20}$

Stein, M. & Smith, M. (1998) Mathematics Teaching in the Middle School. National Council of Teachers of Mathematics

Some qualities of high-level tasks

- Non-routine
- Allow for student reflection
- Allow students to build on their prior knowledge
- Multiple solution strategies
- Reflect high cognitive demand
- Accessible to a wide range of learners, multiple entry points
- Expose what students know and provide information for next steps
- Encourage creativity and imaginative application of knowledge
- Possibilty of multiple answers

Making Connections to The Standards for Mathematical Practice

Instructions: Solve each pair of tasks. Then determine which Standard for Mathematical Practice best describes the demand that will be placed on the learner while engaged in that task.

Task #1:
9 x 2 = 18
9 x 3 = 27
9 x 4 = 36

What patterns do you notice?
What do you notice about the second factor and the first digit of the product?
What do you notice when you add the two digits of the product?
How could these patterns help you solve: 9 x 8?
Does this pattern continue past 9 x 9?

Task #2:
In a sports center in the Philippines are the world's biggest shoes. Each shoe has a width of 2.37 m and a length of 5.29 m. Approximately how tall would a giant be for the shoes to fit? Explain your solution.

Which task above most relates to SMP #4: Model with Mathematics? Explain.

Which task above most relates to SMP #8: Look for and express regularity in repeated reasoning? Explain.

Making Connections to The Standards for Mathematical Practice

Task #3:
Create and label sorts for the following words: square, trapezoid, hexagon, rectangle, rhombus, triangle, pentagon.

Task #4:
How many dimes are in $1,000?

Which task above most relates to SMP #7: Look for and make use of structure? Explain.

Which task above most relates to SMP #6: Attend to precision? Explain.

Making Connections to The Standards for Mathematical Practice

Task #5:
Ten children went to the movie. How many were girls? How many were boys? Explain your answer. Could there be other answers?

Task #6:
Which is closer to one, 5/4 or 4/5?

Which task above most relates to SMP #1: Make sense of problems and persevere in solving them? Explain.

Which task above most relates to SMP #5: Use appropriate tools strategically? Explain.

Making Connections to The Standards for Mathematical Practice

Task #7:
105 students and 5 chaperones went on the field trip. Each bus held 35 people. How many buses were needed?

Task #8:
Do we need zero in our number system?

Which task above most relates to SMP #2: Reason Abstractly and Quantitatively? Explain.

Which task above most relates to SMP #3: Construct viable arguments and critique the reasoning of others? Explain.

Claims for the Mathematics Summative Assessment

Overall Claim for Grades 3-8 "Students can demonstrate progress toward college and career readiness in mathematics."
Overall Claim for Grade 11 "Students can demonstrate college and career readiness in mathematics."

Claim #1 - Concepts & Procedures "Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."
Claim #2 - Problem Solving "Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies."
Claim #3 - Communicating Reasoning "Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."
Claim #4 - Modeling and Data Analysis "Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

notes

Rich, Worthwhile Tasks

Resources for rich tasks:

Illustrative Mathematics
www.illustrativemathematics.org

California Mathematics Framework
http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp

Inside Mathematics
http://www.insidemathematics.org/index.php/mathematical-content-standards

How to modify tasks:

Provide a worthwhile context

Open-up the task to ensure multiple entry points and solution strategies

Eliminate unnecessary support

Reverse the question

Change the position of the unknown

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Your modified task:

Going Forward:

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References:

Richardson, Kathy. *Developing Number Concepts.* Parsippany, NJ: Dale Seymour Publications, 1999.

O'Connel, Susan, and John San Giovanni. *Putting the Practices Into Action: Implementing the Common Core Standards for Mathematical Practice K-8.* Portsmouth, NH: Heinemann, 2013.

Larson, Matthew R., Francis (Skip) Fennell, Thomasenia Lott Adams, Juli K. Dixon, Beth McCord Kobett. *Common Core Mathematics in a PLC at Work.* Bloomington, IN: Heinemann, 2012.

Carpenter, Thomas P., Elizabeth Fennema, Megan Loef Franke, Linda Levi, Susan B. Empson. Children's Mathematics - *Cognitively Guided Instruction.* Portsmouth, NH: Heinemann, 1999.

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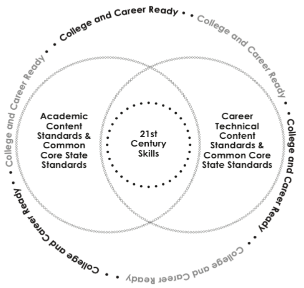

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 +7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5–3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.